

# Distributed MAP Inference for Undirected Graphical Models

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# Motivation

- Graphical models are used in a number of information extraction tasks
- Recently, models are getting larger and denser
  - Coreference Resolution [CULOTTA ET AL. NAACL 2007]
  - Relation Extraction [RIEDEL ET AL. EMNLP 2010, POON & DOMINGOS EMNLP 2009]
  - Joint Inference [FINKEL & MANNING. NAACL 2009, SINGH ET AL. ECML 2009]
- Inference is difficult, and approximations have been proposed
  - LP-Relaxations [MARTINS ET AL. EMNLP 2010]
  - Dual Decomposition [RUSH ET AL. EMNLP 2010]
  - MCMC-Based [MCCALLUM ET AL. NIPS 2009, POON ET AL. AAAI 2008]

**Without parallelization, these approaches have restricted scalability**

# Motivation

Contributions:

- ① Distribute MAP Inference for a large, dense factor graph
  - 1 million variables, 250 machines
- ② Incorporate [sharding](#) as variables in the model

# Outline

## ① Model and Inference

- Graphical Models

- MAP Inference

- Distributed Inference

## ② Cross-Document Coreference

- Coreference Problem

- Pairwise Model

- Inference and Distribution

## ③ Hierarchical Models

- Sub-Entities

- Super-Entities

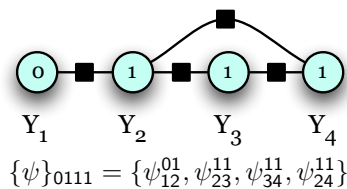
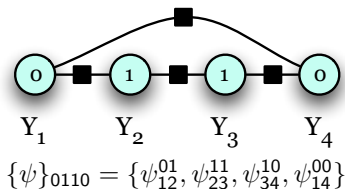
## ④ Large-Scale Experiments

# Factor Graphs

Represent distribution over variables  $Y$  using factors  $\psi$ .

$$p(Y = y) \propto \exp \sum_{y_c \subseteq Y} \psi_c(y_c)$$

**Note:** Set of factors is different of every assignment  $Y = y$  ( $\{\psi\}_y$ )



# MAP<sup>1</sup> Inference

We want to find the **best** configuration according to the model,

$$\begin{aligned}\hat{y} &= \arg \max_y p(Y = y) \\ &= \arg \max_y \exp \sum_{y_c \subseteq y} \psi_c(y_c)\end{aligned}$$

Computational bottlenecks:

- ① Space of  $Y$  is usually enormous (exponential)
- ② Even evaluating  $\sum_{y_c \subseteq y} \psi_c(y_c)$  for each  $y$  may be polynomial

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<sup>1</sup>MAP = maximum a posteriori

# MCMC for MAP Inference

Initial Configuration  $y = y_0$

for (num\_samples):

- ➊ **Propose** a change to  $y$  to get configuration  $y'$   
(Usually a *small* change)
- ➋ Acceptance probability:  $\alpha(y, y') = \min \left( 1, \left( \frac{p(y')}{p(y)} \right)^{1/t} \right)$   
(Only involve computations local to the change)
- ➌ if Toss( $\alpha$ ): **Accept** the change,  $y = y'$

return  $y$

$$\frac{p(y')}{p(y)} = \exp \left\{ \sum_{y'_c \subseteq y'} \psi_c(y'_c) - \sum_{y_c \subseteq y} \psi_c(y_c) \right\}$$

# Mutually Exclusive Proposals

Let  $\{\psi\}_y^{y'}$  be the set of factors used to evaluate a proposal  $y \rightarrow y'$

$$\text{i.e. } \{\psi\}_y^{y'} = (\{\psi\}_y \cup \{\psi\}_{y'}) - (\{\psi\}_y \cap \{\psi\}_{y'})$$

Consider two proposals  $y \rightarrow y_a$  and  $y \rightarrow y_b$  such that,

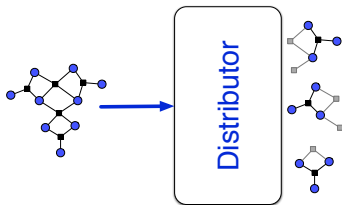
$$\{\psi\}_y^{y_a} \cap \{\psi\}_y^{y_b} = \{\}$$

Completely different set of factors are required to evaluate these proposals.

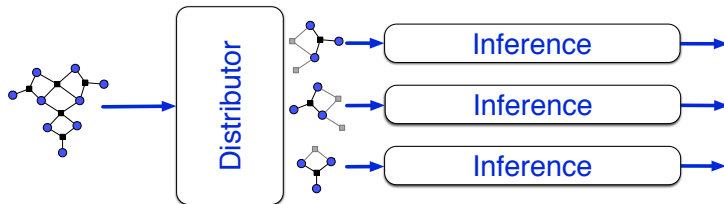
**These two proposals can be evaluated (and accepted) in parallel.**



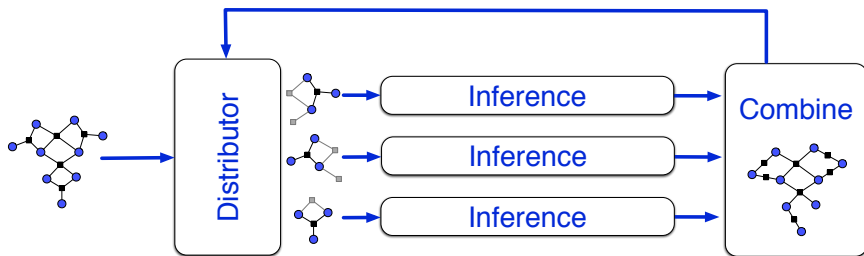
# Distributed Inference



# Distributed Inference



# Distributed Inference



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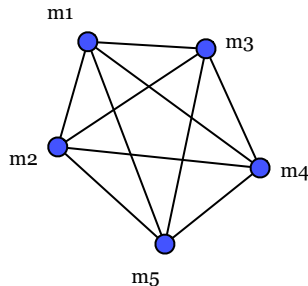
## ③ Hierarchical Models

Sub-Entities

Super-Entities

## ④ Large-Scale Experiments

# Input Features



Define similarity between mentions,  $\phi : \mathcal{M}^2 \rightarrow \mathcal{R}$

- $\phi(m_i, m_j) > 0$ :  $m_i, m_j$  are similar
- $\phi(m_i, m_j) < 0$ :  $m_i, m_j$  are dissimilar

We use cosine similarity of the context bag of words:

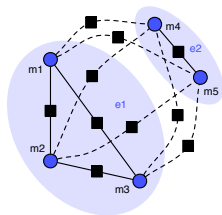
$$\phi(m_i, m_j) = \text{cosSim}(\{c\}_i, \{c\}_j) - b$$

# Graphical Model

The random variables in our model are entities ( $E$ ) and mentions ( $M$ )  
 For any assignment to these entities ( $E = e$ ), we define the model score:

$$p(E = e) \propto \exp \left\{ \sum_{m_i \sim m_j} \psi_a(m_i, m_j) + \sum_{m_i \approx m_j} \psi_r(m_i, m_j) \right\}$$

where  $\psi_a(m_i, m_j) = w_a \phi(m_i, m_j)$ , and  
 $\psi_r(m_i, m_j) = -w_r \phi(m_i, m_j)$

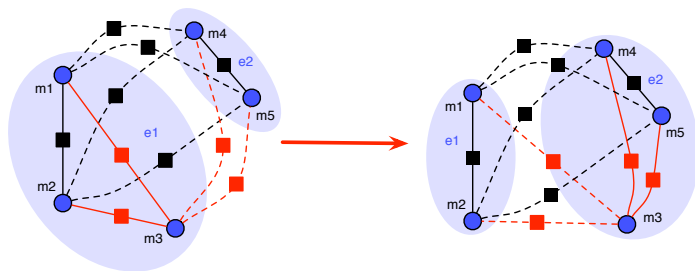


For the following configuration,

$$p(e_1, e_2) \propto \exp \left\{ \begin{aligned} &w_a (\phi_{12} + \phi_{13} + \phi_{23} + \phi_{45}) \\ &- w_r (\phi_{15} + \phi_{25} + \phi_{35} \\ &\quad + \phi_{14} + \phi_{24} + \phi_{34}) \end{aligned} \right\}$$

- 1 Space of  $E$  is Bell Number( $n$ ) in number of mentions
- 2 Evaluating model score for each  $E = e$  is  $O(n^2)$

# MCMC for MAP Inference

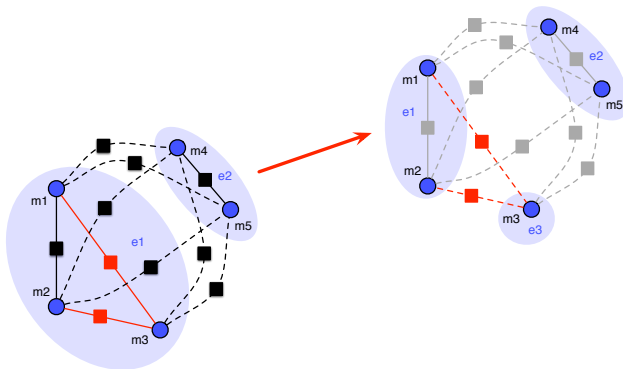


$$p(e) \propto \exp\{w_a(\phi_{12} + \phi_{13} + \phi_{23} + \phi_{45}) - w_r(\phi_{15} + \phi_{25} + \phi_{35} + \phi_{14} + \phi_{24} + \phi_{34})\}$$

$$p(\acute{e}) \propto \exp\{w_a(\phi_{12} + \phi_{34} + \phi_{35} + \phi_{45}) - w_r(\phi_{15} + \phi_{25} + \phi_{13} + \phi_{14} + \phi_{24} + \phi_{23})\}$$

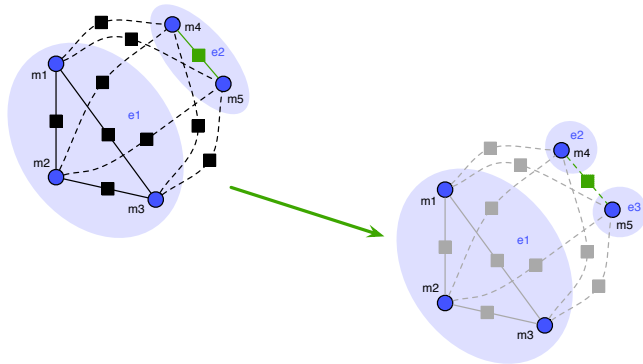
$$\log \frac{p(\acute{e})}{p(e)} = w_a(\phi_{34} + \phi_{35} - \phi_{13} - \phi_{23}) - w_r(\phi_{13} + \phi_{23} - \phi_{34} - \phi_{35})$$

# Mutually Exclusive Proposals

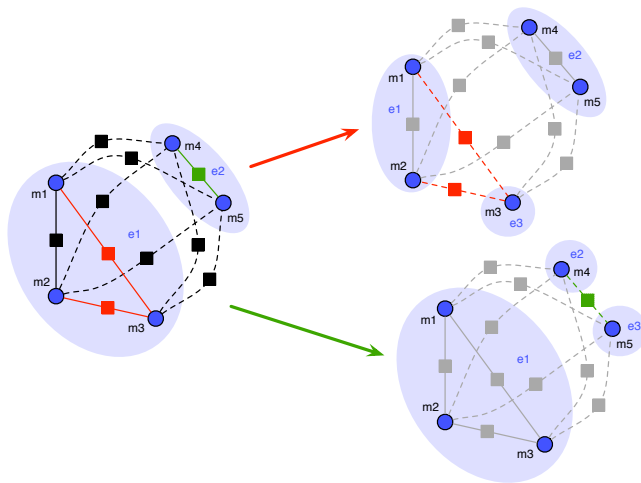




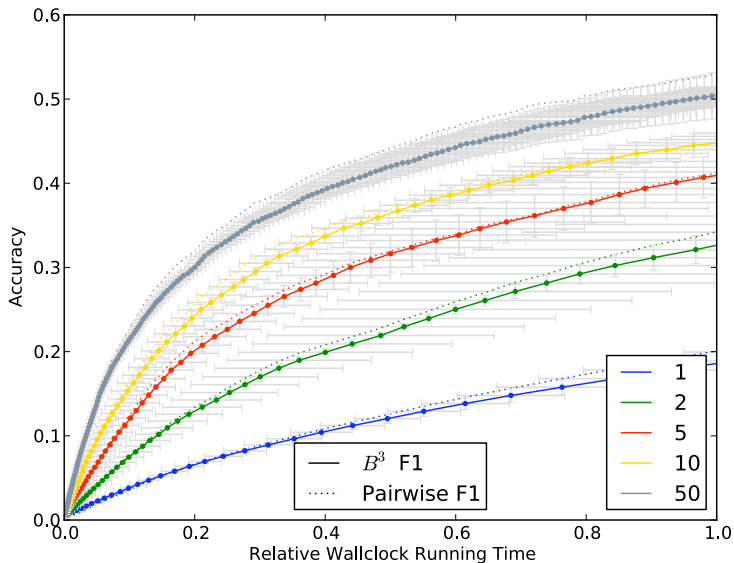
# Mutually Exclusive Proposals



# Mutually Exclusive Proposals



# Results



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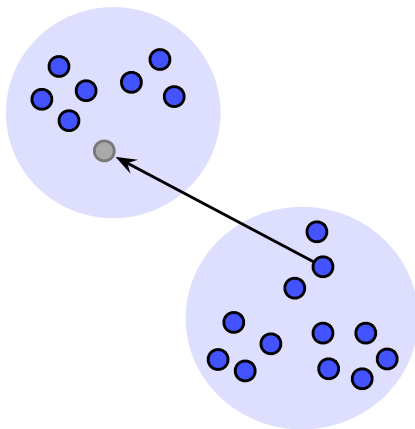
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Sub-Entities

Super-Entities

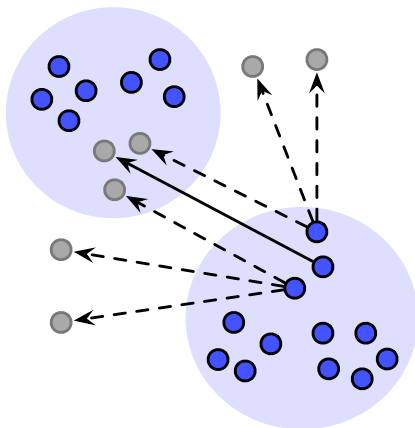
## ④ Large-Scale Experiments

# Sub-Entities



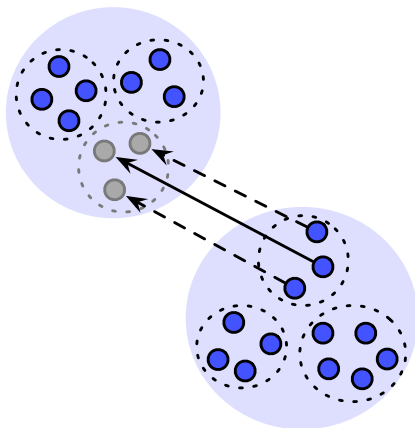
- Consider an **accepted** move for a mention

# Sub-Entities



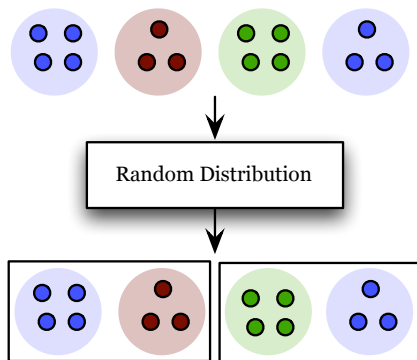
- Ideally, *similar* mentions should also move to the same entity
- Default proposal function does not utilize this
- *Good* proposals become more rare with larger datasets

# Sub-Entities



- Include **Sub-Entity** variables
- Model score is used to sample sub-entity variables
- Propose moves of mentions in a sub-entity simultaneously

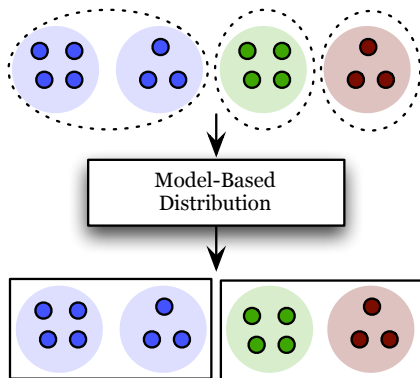
# Super-Entities



- Random distribution may not assign *similar* entities to the same machine
- Probability that similar entities will be assigned to the same machine is small

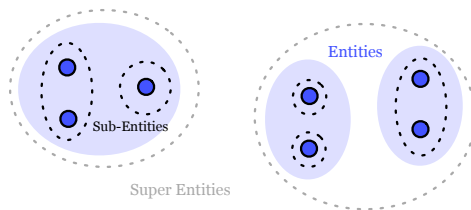


# Super-Entities



- Augment model with **Super-Entities** variables
- Entities in the same super-entity are assigned the same machine
- Model score is used to sample super-entity variables

# Hierarchical Representation

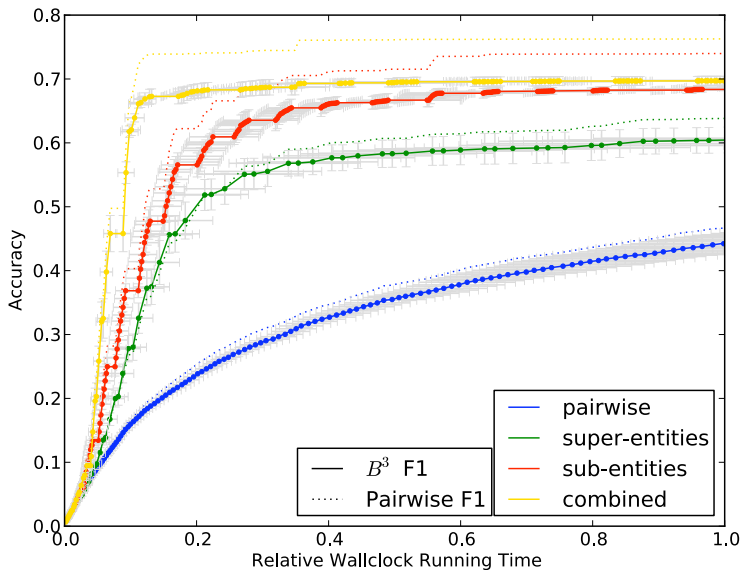


## • Factors

- Affinity factors between **mentions** **sub-entities** in the same **sub-entities** **entities** **super-entities**
- Repulsion factors are similarly symmetric across levels

- **Sampling:** Fix variables of two levels, sample the remaining level

# Evaluation



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# Preliminary Large-Scale Experiments

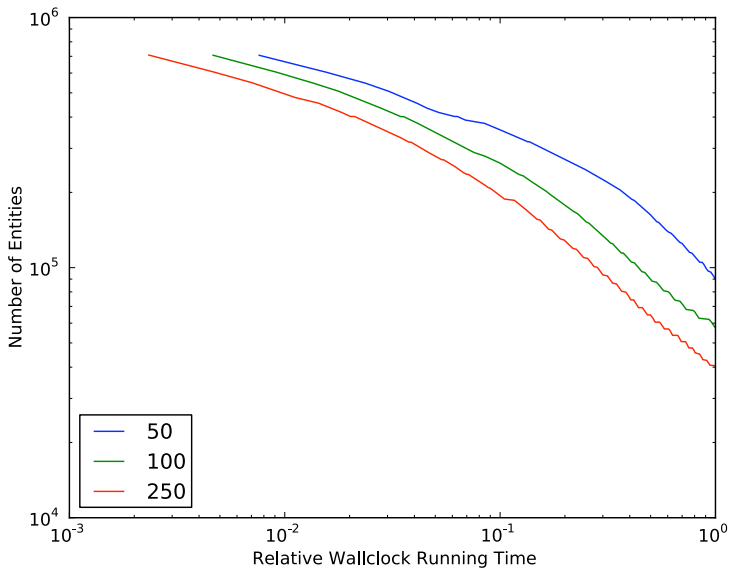
## Data

- *New York Times Annotated Corpus* [SANDHUS LDC 2008]  
20 years of articles (1987-2007)
- prune rare names ( $<1000$ ):  $\sim 1$  million **person name** mentions

## Evaluation

- Automated labels are too noisy for evaluation
- Instead, we estimate the **speed of inference**
  - trust the model to accept good proposals
  - observe the number of predicted entities

# Speed of Inference



# Related Work

- GraphLab [LOW ET AL. UAI 2010]
  - how do we represent dynamic graphs
  - how do we represent hierarchical models
- Graph Splashing [GONZALEZ ET AL. UAI 2009]
  - graph structure changes with every configuration
  - BP messages are enormous for exponential-domain variables
- Topic Models [SMOLA & NARAYANMURTHY. VLDB 2010, ASUNCION ET AL. NIPS 2009]
  - restrictions since they are calculating probabilities
  - we allow non-random distribution and customized proposals

# Conclusions

- 1 propose [distributed inference](#) for graphical models
- 2 enable distributed [cross-document coreference](#)
- 3 improve sharding with latent [hierarchical](#) variables
- 4 demonstrate utility on [large](#) datasets

## Future Work:

- more [scalability](#) experiments
- study [mixing](#) and [convergence](#) properties
- add more expressive [factors](#)
- [supervision](#): labeled data, noisy evidences



# Thanks!

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